sure to bid again and improve his payoff, for

$$(1 - \lambda)(1 - x) > (1 - \lambda)\left(1 - \frac{2\lambda}{1 + \lambda}\right) = \frac{(1 - \lambda)^2}{1 + \lambda} = \frac{\lambda}{1 + \lambda};$$

while if A_1 chooses to move, he is assured of getting this much himself. In either case the payoff to A_3 will be decreased. Thus, if A_2 bids $\lambda/1+\lambda$ then A_3 will follow with $2\lambda/1+\lambda$ and terminate the bidding. The natural question to ask is: can A_2 improve his payoff by bidding $x<(\lambda/1+\lambda)$? (Clearly, $x>(\lambda/1+\lambda)$ will not do.) If A_3 still does best by terminating the bidding, he has to move to the right, thus widening A_2's payoff even further. It is conjectured that A_3 will choose that strategy as long as he can not get more by the same scheme on the interval $1-x$. The value of x can now be evaluated by solving the equation

$$(1 - x)\left(1 - x - \frac{x}{1 - \lambda}\right) = \frac{x}{1 - \lambda}.$$

The solution is given by $\mu=1-\sqrt{(1-\lambda)}$. Thus, it is conjectured that the 3-player game will proceed as follows: (a) A_1 bids 0; (b) A_2 bids $\mu\cong.214$; (c) A_3 bids $1-(\mu/1-\lambda)\cong.654$; (d) Everyone passes. This means that the second player is the most powerful, and his payoff is about 44 percent of the whole interval.

It is conjectured that the n-player game proceeds in a similar manner; namely, that each player bids only once. Let $a_1^n, a_2^n, \cdots, a_n^n$ be the bids made by $A_1, A_2, \cdots, A_n$. Clearly, $a_1^n=0$. It is conjectured that

$$\begin{aligned}
a_2^{n+1} &= a_2^n(1 - a_3^{n+1} + a_2^{n+1}) \\
a_4^{n+1} - a_3^{n+1} &= (a_3^{n+1} - a_2^{n+1})(1 - a_2^{n+1}) \\
a_5^{n+1} - a_4^{n+1} &= (a_4^n - a_3^n)(1 - a_3^{n+1} + a_2^{n+1}) \qquad (\text{define } a_4^3 = 1) \\
&\cdots\cdots\cdots\cdots\cdots \\
a_{n+1}^{n+1} - a_n^{n+1} &= (a_n^n - a_{n-1}^n)(1 - a_3^{n+1} + a_2^{n+1}) \\
1 - a_{n+1}^{n+1} &= (1 - a_n^n)(1 - a_3^{n+1} + a_2^{n+1})
\end{aligned}$$

describe the solution for the $(n+1)$-player game, where for $n=2$ the first two equations are used instead of the first and the last.

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CONVERGENCE ALMOST EVERYWHERE IS NOT TOPOLOGICAL

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When the notion of convergence almost everywhere (a.e.) is introduced, the student acquainted with convergence in general topology may try to carry over more properties than are justified. This note shows that if X is the space of bounded measurable real functions on $[0, 1]$, convergence a.e. is not equivalent to convergence in any topology on X.

Let $\{f_n\}$ be a sequence of functions which converges in measure to zero but fails to converge a.e.; the standard example is the sequence $f_1^1, f_1^2, f_2^2, f_1^3, f_2^3, \cdots$, where

$$f_m^n(x) = \begin{cases} 1 & (m-1)/n \leqq x \leqq m/n \\ 0 & \text{otherwise} \end{cases} \qquad 1 \leqq m \leqq n.$$

Now suppose that a topology of convergence a.e. exists on X. Since $\{f_n\}$ fails to converge to zero, there must be a neighborhood $N(0)$ which f_n is frequently outside; let $\{f_{n'}\}$ be the subsequence of terms outside of $N(0)$. Then $\{f_{n'}\}$ converges in measure to zero, so by a standard theorem ([**1**], p. 46), it has a subsequence which converges a.e. to zero. But that subsequence is eventually in $N(0)$, contradicting the choice of $\{f_{n'}\}$ to remain outside. That is, the supposed topology cannot exist; in fact, there is no way to define convergence a.e. by a neighborhood filter of the usual sort.

Reference

1. A. N. Kolmogorov and S. V. Fomin, Functional Analysis, Vol. 2, Graylock, Albany, 1961.

AN ELEMENTARY CONSTRUCTION OF A FINITE NONARGUESIAN PROJECTIVE PLANE

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1. Introduction. We give here a construction of a known finite nonarguesian plane in such a way that beginning students can easily check the details. Our construction is equivalent to one that is well-known [**1**, p. 408; **2**, p. 364].

In Section 2, a Veblen-Wedderburn system with 9 elements is presented. The corresponding projective plane is given in section 3. Also, we outline six cases in which the theorem of Desargues fails.

The paper can be read easily by a student with an elementary course in abstract algebra, and the author believes the material could be used in connection with an elementary course in algebra or geometry.

2. Construction of a Veblen-Wedderburn system. Let F be the field of nine elements, with the usual identities 0, 1, and with $2=1+1$. (Recall that F can be displayed as the set of elements $m+ni$, where m, n represent residue classes modulo 3. Addition is given by $(m_1+n_1i)+(m_2+n_2i)=(m_1+m_2)+(n_1+n_2)i$, and multiplication makes use of the definition that $i^2+1=0$ [**1**, p. 410].)

An element x of F is called *square* provided $x=a^2$ for some $a \in F$. It is easy to prove the following for $x \in F$:

(2.1) $2x^5+2x=x$ or 0, according to whether x is square or not square. (The square elements are 0, 1, 2, i, and $2i$.)

(2.2) $x^5+2x=0$ or x, according to whether x is square or not square.

(2.3) $x^2+x^4+x^6=2$ for all x except 0, 1, 2.

Next we use the elements of F to form a Veblen-Wedderburn system V.